Total Printed Pages : **16** **Booklet No.** **11199**

# M.Phil./Ph.D. ADMISSION TEST, 2019 & 2020

## Paper II

## Subject : 131 - MATHEMATICS

Roll No. (In figures) ........................................(In words) ........................................

OMR Sheet Barcode No. ........................................

Signatures of Invigilators 1. ........................................ 2. ........................................

Names of Invigilators 1. ........................................ 2. ........................................

Time : 2 Hours Max. Marks : 200

SEAL

## GENERAL INSTRUCTIONS

1. Read the instructions given on the Question Booklet and OMR Sheet before starting the answers. All the entries should be filled by **blue or black ball point pen**.

2. The Question Booklet contains **100** questions and all questions are compulsory.

3. Each question is of **2** marks. There is **no negative marking**.

4. Candidates must ensure that the Question Booklet issued to them has all the questions. Defective Question Booklet can be got changed within **10** minutes.

1. प्रश्नों के उत्तर लिखने से पूर्व प्रश्न-पुस्तिका और ओ.एम.आर. शीट पर दिये हुए निर्देश पढ़ें। सभी प्रविष्टियाँ **नीले अथवा काले बॉल पॉइन्ट पेन** से भरें।

2. प्रश्न-पुस्तिका में **100** प्रश्न हैं और सभी प्रश्न अनिवार्य हैं।

3. प्रत्येक प्रश्न **2** अंक का है। कोई **नकारात्मक अंकन (negative marking) नहीं होगा।**

4. परीक्षार्थी सुनिश्चित कर लें कि उन्हें जो प्रश्न-पुस्तिका दी गई है उसमें सभी प्रश्न अंकित हैं। त्रुटिपूर्ण प्रश्न-पुस्तिका **10** मिनट की अवधि में बदलवाई जा सकती है।

1. If $x$ and $y$ are real numbers then which one of the following is always true ?

(A) $|x-y| \leq |x|-|y|$

(B) $|x-y| \geqslant |x|+|y|$

(C) $|x-y| \geqslant ||x|-|y||$

(D) $|x-y| = |x|-|y|$

2. The set of all real number $x$ such that $| |3 - x| - |x + 2| | = 5$ is :

(A) $[3, \infty)$

(B) $(-\infty, -2]$

(C) $(-\infty, -2] \cup [3, \infty)$

(D) $(-\infty, -3] \cup [2, \infty)$

3. The improper integral $\int_a^\infty \frac{dx}{x^n}$, $(a > 0)$ converges if :

(A) $n > 1$

(B) $n = 1$

(C) $n < 1$

(D) $n = 0$

4. Which of the following sets is uncountable ?

(A) $\{1, 4, 9, 16, 25 ...\}$

(B) $\{2n : n \in N\}$

(C) All rational numbers

(D) All irrational numbers

5. The least upper bound of the set $\left\{\frac{1}{n}, n \in N\right\}$ is :

(A) 1

(B) 0

(C) $-1$

(D) None of these

6. The sequence $<S_n>$ where $S_n = \left(1 + \frac{2}{n}\right)^{n+3}$ converges to :

(A) e

(B) $e^2$

(C) $e+3$

(D) $e^2+3$

7. For the given sequence $\left\langle (-1)^n \left(1 + \frac{1}{n}\right)\right\rangle$ which one of the following statements is **correct** ?

(A) Limit superior = limit inferior

(B) Neither limit superior nor limit inferior exists

(C) Limit superior = 1 and limit inferior = $-1$

(D) Limit superior = 1 and limit inferior = 0

8. Which of the following statements is **true** ?

(A) For any positive number $\in$, there is a natural number n such that $\frac{1}{n} < \in$

(B) Between any two real numbers there is no irrational number

(C) Convergent sequence is not bounded

(D) None of the above is true

9. If $f(x) = x^2$ for all $x \in R$, $f$ is :

(A) not continuous on R

(B) uniformly continuous on R

(C) not uniformly continuous on R

(D) none of these

10. Consider the following functions :

(1) $y = x \sin\left(\frac{1}{x}\right)$ if $x \neq 0$
$\quad 0$ if $x = 0$

(2) $y = x^2 \sin\left(\frac{1}{x}\right)$ if $x \neq 0$
$\quad 0$ if $x = 0$

(3) $y = x^2 \cos\left(\frac{1}{x}\right)$ if $x \neq 0$
$\quad 0$ if $x = 0$

(4) $y = x \cos\left(\frac{1}{x}\right)$ if $x \neq 0$
$\quad 0$ if $x = 0$

The functions differentiable at $x = 0$ are :

(A) (1) and (2)

(B) (2) and (3)

(C) (3) and (4)

(D) (1) and (4)

19. The numbers 2697, 2759, 2821 and 2883 are divisible by 31. The determinant $\begin{bmatrix} 2 & 6 & 9 & 7 \\ 2 & 7 & 5 & 9 \\ 2 & 8 & 2 & 1 \\ 2 & 8 & 8 & 3 \end{bmatrix}$ is divisible by :

(A) 31

(B) 13

(C) 26

(D) 62

20. If for the matrix A, $A^3 = I$ where I is the identity matrix then $A^{-1}$ is :

(A) $A^2$

(B) $A^3$

(C) A

(D) None of these

21. Let U and V be two vector spaces over a field F of dimensions m and n respectively. Then Hom (U, V) (the set of all linear mappings of U and V) is a vector space over F of dimension.

(A) $m^n$

(B) $n^m$

(C) mn

(D) m+n

22. Let AX = B represent a non-homogeneous system of 3 linear equations in 3 unknowns. Then which of the following is **true** ?

(A) The system always has a unique solution.

(B) The system always has infinitely many solutions.

(C) If $|A| = 0$ and (adj A)B = 0 then the given system of equations is inconsistent and has no solution.

(D) The system has a unique solution if and only if $|A| \neq 0$ and $X = \left(\frac{1}{|A|} \text{adj } A\right)B$.

23. In a real quadratic form, the matrix associated with the real quadratic form is a real :

(A) symmetric matrix

(B) skew - symmetric matrix

(C) singular matrix

(D) non - singular matrix

24. Let $T : U \to V$ be a linear mapping and rank T = n then :

(A) dim Ker T = n

(B) dim Im T = n

(C) dim V = n

(D) dim U = n

25. Let U and V be two vector spaces of dimension m and n respectively. Let $T : U \to V$ be a linear transformation of rank r. Then the nullity of T is :

(A) m − r

(B) n − r

(C) m + n − r

(D) mn − r

26. A linear mapping $T : U \to V$ is injective if and only if :

(A) T is surjective

(B) Ker T = {0}

(C) Im T = {0}

(D) None of these

27. Consider the set of vectors

$S_1 = \{(3, 0, 4), (-4, 0, 3), (0, 9, 0)\}$

$S_2 = \left\{\left(\frac{3}{5}, 0, \frac{4}{5}\right), \left(\frac{-4}{5}, 0, \frac{3}{5}\right), (0, 1, 0)\right\}$

(A) $S_1$ is orthogonal and $S_2$ is orthonormal

(B) $S_1$ is orthonormal and $S_2$ is not orthonormal

(C) Both $S_1$ and $S_2$ are not orthogonal

(D) Both $S_1$ and $S_2$ are orthonormal

36. Let $K = Q(\sqrt{2}, \sqrt{6})$ and $F = Q$ be two fields. Then degree of extension of K over F is :

(A) 6

(B) 4

(C) 2

(D) 3

37. Let A be a square matrix of order 3. Then, rank(A) = 3, only if :

(A) there are 3 distinct eigen values of A

(B) $\det(A) \neq 0$

(C) all eigen values are non-zero

(D) all of these

38. Let R[x] be the polynomial ring over field of real numbers and $p(x) = x^2 + 1$. Then :

(A) $\frac{R[x]}{< p(x) >}$ is an integral domain but not a field

(B) $\frac{R[x]}{< p(x) >}$ is a division ring but not a field

(C) $\frac{R[x]}{< p(x) >}$ is a Euclidean domain but not a field

(D) $\frac{R[x]}{< p(x) >}$ is a field

39. Let $p(x) = x^3 - 1$ be a polynomial over Q, the field of rational numbers. Then the degree of the splitting field of p(x) over Q is :

(A) 2

(B) 3

(C) 4

(D) 6

40. The order of Galois group of the splitting field of the polynomial $p(x) = x^4 + 1$ over Q is :

(A) 2

(B) 4

(C) 6

(D) 8

41. Let $f : Q \to Q$ be a function defined as $f(x) = x^{-1}$. Then.

(A) $f$ is a homomorphism

(B) $f$ is a monomorphism

(C) $f$ is an endomorphism

(D) $f$ is an automorphism

42. Let G be a finite abelian group. Then G is :

(A) isomorphic to the direct product of its Sylow subgroups

(B) homomorphic to the direct product of its Sylow subgroups

(C) isomorphic to the direct sum of its Sylow subgroups

(D) homomorphic to the direct sum of its Sylow subgroups

43. Let G be a group of order 24. Then

(A) G has a subgroup of order 6

(B) G has a subgroup of order 12

(C) G has a subgroup of order 8

(D) G has a subgroup of order 10

44. Let $K_4 = \{4n : n \in Z\}$, where $(Z, +, \cdot)$ is the ring of integers. Then

(A) $K_4$ is an ideal of Z but $Z/K_4$ is not a factor ring

(B) $K_4$ is an ideal of Z but $Z/K_4$ is a factor ring

(C) $K_4$ is a subring but not an ideal of Z

(D) $K_4$ is an ideal but not a subring

45. Let $p_A(x)$ denote the characteristic polynomial of the matrix A. Then, for which of the following matrices, $p_A(x) - p_{A^{-1}}(x)$ is a constant ?

(A) $\begin{bmatrix} 3 & 3 \\ 2 & 4 \end{bmatrix}$

(B) $\begin{bmatrix} 4 & 3 \\ 2 & 3 \end{bmatrix}$

(C) $\begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}$

(D) $\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$

57. Let $R_1$ and $R_2$ be the radii of convergence of the power series $\sum_{n=1}^{\infty} a_n z^n$ and $\sum_{n=1}^{\infty} n a_n z^{n-1}$, respectively. Then which of the following is **correct** ?

(A) $R_1 < R_2$

(B) $R_1 > R_2$

(C) $R_1 = R_2$

(D) Cannot determine

58. The residue of the function $f(z) = \dfrac{2z}{(z+4)(z-1)^2}$ at pole $z = 1$ is :

(A) 1/5

(B) 2/5

(C) 8/25

(D) 4/25

59. The mobius transformation takes

(A) Circle into line

(B) Line into circle

(C) Circle into square

(D) Circle into circle

60. The function $f(z) = \cos(2z+1)$ is :

(A) unbounded and nowhere analytic

(B) bounded and entire

(C) bounded but nowhere analytic

(D) unbounded and entire

61. A complex number is said to be algebraic number if it is algebraic over field of :

(A) Real numbers

(B) Complex numbers

(C) Irrational numbers

(D) Rational numbers

62. Let $f(z) = u(r, \theta) + iv(r, \theta)$ be an analytic function, then :

(A) $\dfrac{\partial u}{\partial r} = r\dfrac{\partial v}{\partial \theta}, \dfrac{\partial u}{\partial r} = -\dfrac{1}{r}\dfrac{\partial v}{\partial \theta}$

(B) $\dfrac{\partial u}{\partial r} = \dfrac{1}{r}\dfrac{\partial v}{\partial \theta}, \dfrac{\partial u}{\partial \theta} = -r\dfrac{\partial v}{\partial r}$

(C) $\dfrac{\partial u}{\partial r} = r\dfrac{\partial v}{\partial \theta}, \dfrac{\partial u}{\partial \theta} = -r\dfrac{\partial v}{\partial r}$

(D) None of these

63. Let the points $z_1$, $z_2$ and $z_3$ are lying on the same line, then which of the following options is **correct** ?

(A) $\arg\left(\dfrac{z_1 - z_2}{z_3 - z_2}\right) = \dfrac{2\pi}{3}$

(B) $\arg\left(\dfrac{z_1 - z_2}{z_3 - z_2}\right) = 0$

(C) $\arg\left(\dfrac{z_1 - z_2}{z_3 - z_2}\right) = \dfrac{\pi}{3}$

(D) $\arg\left(\dfrac{z_1 - z_2}{z_3 - z_2}\right) = \dfrac{\pi}{2}$

64. The series $f(z) = \sum_{n=-\infty}^{\infty} a_n (z - z_0)^n$ where $a_n = \dfrac{1}{2\pi i} \int_C \dfrac{f(z)}{(z - z_0)^{n+1}} dz$ represents :

(A) Laurent

(B) Taylor

(C) Maclaurin

(D) None of these

72. The covariant derivative of an invariant is :

(A) Constant but not equal to zero

(B) A contravariant vector

(C) A covariant vector

(D) The same as its ordinary derivative

73. The integral surface of the partial differential equation $yp - xq = 0$ with Cauchy data $x = 0, z = y^2$, (where the symbols have their usual meanings) is :

(A) $x^2 + y^2 = z^2$

(B) $x = y^2 + z^2$

(C) $y = x^2 + z^2$

(D) $z = x^2 + y^2$

74. The partial differential equation $x\dfrac{\partial^2 u}{\partial x^2} + y\dfrac{\partial^2 u}{\partial y^2} = 0$ is :

(A) Elliptic for $x > 0, y < 0$

(B) Elliptic for $x < 0, y > 0$

(C) Hyperbolic for $x > 0, y < 0$

(D) Hyperbolic for $x > 0, y > 0$

75. If $u(x, t)$ be a solution of the heat equation $\dfrac{\partial^2 u}{\partial x^2} = \dfrac{1}{C}\dfrac{\partial u}{\partial t}$, then :

(A) $u(x, t) = \dfrac{1}{\sqrt{t}} \exp\left(-\dfrac{x^2}{4ct}\right)$

(B) $u(x, t) = \dfrac{1}{\sqrt{t}} \exp\left(-\dfrac{x}{4ct}\right)$

(C) $u(x, t) = \dfrac{1}{\sqrt{t}} \exp\left(-\dfrac{x^2}{4c\sqrt{t}}\right)$

(D) $u(x, t) = \dfrac{1}{t} \exp\left(-\dfrac{x}{4ct}\right)$

76. Newton - Raphson iteration formula for finding $k^{th}$ root of the number N, where $N > 0$ is :

(A) $x_{n+1} = \dfrac{(k-1)\, x_n^k + \sqrt[k]{N}}{(k-1)\, x_n^{k-1}}, n = 0, 1, 2, 3, \ldots$

(B) $x_{n+1} = \dfrac{k\, x_n^k - \sqrt[k]{N}}{k\, x_n^{k-1}}, n = 0, 1, 2, 3, \ldots$

(C) $x_{n+1} = \dfrac{k\, x_n^k + N}{k\, x_n^{k-1}}, n = 0, 1, 2, 3, \ldots$

(D) $x_{n+1} = \dfrac{(k-1)\, x_n^k + N}{k\, x_n^{k-1}}, n = 0, 1, 2, 3, \ldots$

77. By Gauss elimination method, the solution of the system of equations :

$3x + 4y + 5z = 40$

$2x - 3y + 4z = 13$

$x + y + z = 9$

is :

(A) $x = 6, y = 3, z = 2$

(B) $x = 4, y = 2, z = 4$

(C) $x = 1, y = 3, z = 5$

(D) $x = 2, y = 1, z = 6$

78. A polynomial satisfied by (0, 1), (1, 1), (2, 2) and (4, 5) is given by :

(A) $\dfrac{1}{6}\left(-x^3 + 9x^2 - 8x + 6\right)$

(B) $\dfrac{1}{9}\left(-x^3 + 9x^2 - 8x + 9\right)$

(C) $\dfrac{1}{12}\left(-x^3 + 9x^2 - 8x + 12\right)$

(D) $x^3 - 9x^2 + 8x + 1$

85. The Kernel $K(x, t)$ of the Fredholm integral equation of the second kind, is separable (or degenerate), if :

(A) $K(x, t) = \sum_{i=1}^{n} g_i(x)h_i(t)$

(B) $K(x, t) = \sum_{i=1}^{\infty} g_i(x)h_i(t)$

(C) $K(x, t) = \sum_{i=1}^{n} g_i(x, t)h_i(x, t)$

(D) $K(x, t) = \sum_{i=1}^{\infty} g_i(x, t)h_i(x, t)$

86. Given that the eigen values of the integral equation $y(x) = \lambda \int_0^{2\pi} \cos(x + t)\, y(t)dt$ are $\frac{1}{\pi}$ and $-\frac{1}{\pi}$ with respect to eigen functions $\cos x$ and $\sin x$, then, the integral equation $y(x) = \sin x + \cos x + \lambda \int_0^{2\pi} \cos(x + t)\, y(t)$ has :

(A) Unique solution for $\lambda = -\frac{1}{\pi}$

(B) Unique solution for $\lambda = \frac{1}{\pi}$

(C) Unique solution for $\lambda = -\pi$

(D) No solution for $\lambda = -\pi$

87. Let $u(x, t)$ be a solution of the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ in a rectangle $[0, \pi] \times [0, T]$ subject to the boundary conditions $u(0, t) = u(\pi, t) = 0$, $0 \le t \le T$ and the initial condition $u(x, 0) = \phi(x)$, $0 \le x \le \pi$. If $f(x) = u(x, T)$, then which of the following is true for a suitable kernel $k(x, y)$ ?

(A) $f(x) = \int_0^{\pi} k(x, y)\, \phi(y)dy, 0 \le x \le \pi$

(B) $f(x) = \phi(x) + \int_0^{\pi} k(x, y)\, \phi(y)dy, 0 \le x \le \pi$

(C) $f(x) = \int_0^{x} k(x, y)\, \phi(y)dy, 0 \le x \le \pi$

(D) $f(x) = \phi(x) + \int_0^{x} k(x, y)\, \phi(y)dy, 0 \le x \le \pi$

88. The Lagrange's equations of motion for a conservative holonomic dynamical system in terms of Lagrangian function L (if $q_r$, $r = 1, 2, \dots n$ are generalised coordinates) are :

(A) $\frac{\partial L}{\partial \dot{q}_r} = \frac{\partial L}{q_r}, r = 1, 2, 3, \dots n$

(B) $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_r}\right) = \frac{\partial L}{\partial q_r}, r = 1, 2, 3, \dots n$

(C) $\frac{\partial^2 L}{\partial \dot{q}_r^2} = \frac{\partial L}{\partial q_r}, r = 1, 2, 3, \dots n$

(D) $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_r}\right) = \frac{\partial L}{\partial \dot{q}_r}, r = 1, 2, 3, \dots n$

95. Dynamic programming deals with the :

(A) Single - stage decision making problems

(B) Multi - stage decision making problems

(C) Problems which fix the levels of different variables so as to maximize profit or minimize cost

(D) Time - independent decision making problems

96. With the standard/usual notations, which one is the equation of osculating plane in vector notation ?

(A) $\left[\vec{R} - \vec{r}(t), \vec{r}, \dot{\vec{r}}\right] = 0$

(B) $\left[\vec{R} - \vec{r}(t), \dot{\vec{r}}, \dot{\vec{r}}\right] = 0$

(C) $\left[R - \vec{r}(t), \vec{r}, \ddot{\vec{r}}\right] = 0$

(D) $\left[R - \vec{r}(t), \dot{\vec{r}}, \ddot{\vec{r}}\right] = 0$

97. If ρ denotes the radius of curvature and the other symbols have their usual meanings, then which one relation is **true** ?

(A) $\frac{1}{\rho^2} = \frac{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}{\dot{s}^2}$

(B) $\frac{1}{\rho^2} = \frac{\dot{x}^2 + \dot{y}^2 + \dot{z}^2 - \dot{s}^2}{\dot{s}^4}$

(C) $\frac{1}{\rho^2} = \frac{\ddot{x}^2 + \ddot{y}^2 + \ddot{z}^2}{\dot{s}^2}$

(D) $\frac{1}{\rho^2} = \frac{\ddot{x}^2 + \ddot{y}^2 + \ddot{z}^2 - \ddot{s}^2}{\dot{s}^4}$

98. If $g_{ij}$ and $a_{ij}$ are components of two symmetric covariant tensors and $\begin{Bmatrix} i \\ jk \end{Bmatrix}_g$, $\begin{Bmatrix} i \\ jk \end{Bmatrix}_a$ are the corresponding Christoffel symbols of the second kind then the entity $\begin{Bmatrix} i \\ jk \end{Bmatrix}_g - \begin{Bmatrix} i \\ jk \end{Bmatrix}_a$ represents components of :

(A) A covariant tensor of first order

(B) A contravariant tensor of second order

(C) A mixed tensor

(D) A non - tensor entity

99. If $L^{-1}$ is the inverse Laplace transformation operator, then $L^{-1}\left\{\frac{s}{2s^2 - 8}\right\}$ is equal to :

(A) $\frac{1}{2}$ cos h2t

(B) $\frac{1}{2}$ cos ht

(C) cos h $\left(\frac{t}{2}\right)$

(D) $\frac{1}{2}$ sin ht

100. The Fourier transform of $f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$ is :

(A) $\frac{1}{s}$ sin sa

(B) $\frac{2}{s}$ sin sa

(C) $\frac{2i}{s^2}$ (cos sa − sin sa)

(D) $\frac{2i}{s}$ (cos sa + sin sa)

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